MATHEMATICAL MODEL OF THE OSCILLATORY CYCLE ASSOCIATED WITH NONSTEADY INTERACTION OF A SUPERSONIC JET WITH A BARRIER

V. G. Dulov

UDC 533.6.011

INTRODUCTION

When a supersonic jet interacts with a barrier, a complex flow pattern is created with a branched system of compression shocks containing a region of local subsonic flow, contact discontinuities, and flow zones with large gradients of the parameters (see Fig. 1, in which the solid lines represent shock fronts and the dashed lines represent contact discontinuities) Numerous experimental data (see, e.g., [1, 2]) indicate that for a certain bounded set of initial parameters a physical phenomenon occurs, which has not been adequately studied in the theoretical aspects, namely, the shock configuration of the jet loses stability, the steady flow near the obstacle changes spontaneously into nonsteady flow, and a self-sustaining strongly fluctuating wave process develops directly in front of the barrier.

The hypothesis has been advanced [3] that a definite role is played in the mechanism of this effect by an internal turbulent wake, which periodically emerges and decays behind the branch line of the shock fronts. This wake is formed when immediately after the triple point there occurs not one, but two contact discontinuities, between which is formed an isobaric region filled with gas in the rest state relative to the shock branch point.

Under steady-state conditions a wake cannot occur after the triple point, and in this sense the fact that it is there may be regarded as the primary cause of the transition from steady to nonsteady flow. In the present article we propose a mathematical model of this effect, based on the hypothesis of a periodically emerging and decaying wake.

The flow fluctuations are accompanied by appreciable displacements realtive to the nonuniform background of the strong central shock, whereupon strong entropy waves are transmitted through the postshock jet stream. Here we witness a phenomenon analogous to a nonsteady entropy layer.

Several discrete oscillatory tones are usually observed, differing considerably in frequency. The low-frequency fluctuations can have a very large amplitude and are of primary concern in the investigation. The high-frequency amplitudes are small, and frequency estimates show that these oscillations are associated with processes propagating with the speed of sound.

The first step in construction of a model of the low-frequency cycles is to disregard the role of high-frequency oscillations and their distorting influence on the evolution of all processes with time. This condition is realized when the sound velocity is considered to be infinite in the subsonic region after the central shock. In this approximation only entropy waves upset the quasisteady state of the process in the subsonic region.

§1. The analytical description of the entropy waves rests on the following assumptions. The relatively small region of essentially three-dimensional flow in front of the barrier is treated in the integral aspect as a simulation discontinuity on the basis of general conservation laws [4]. Then in the flow zone between the central compression shock and cross section 1—1 (Fig. 2) we can use the one-dimensional approximation; between cross sections 1—1 and 2—2 the flow has a distinct three-dimensional behavior and admits quasisteady description by means of general conservation laws. The gas flowing across the central shock moves to cross section 1—1 in a channel with cross-sectional area F, which varies along the length of the channel and with time. The equations of motion of the gas in this channel are written in the form

$$\frac{\partial \rho F}{\partial t}+\frac{\partial v\rho F}{\partial x}=0,\quad \frac{\partial v}{\partial t}+v\,\frac{\partial v}{\partial x}=-\frac{1}{\rho}\,\frac{\partial p}{\partial x},\quad \frac{\partial S}{\partial t}+v\,\frac{\partial S}{\partial x}=0, \tag{1.1}$$

Krasnoyarsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 48-58, November-December, 1978. Original article submitted January 30, 1978.

0021-8944/78/1906-0747$07.50

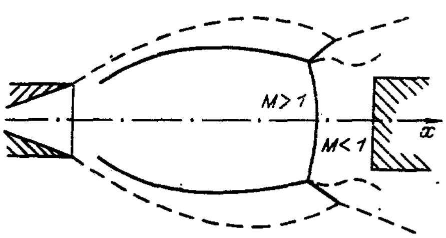

Fig. 1

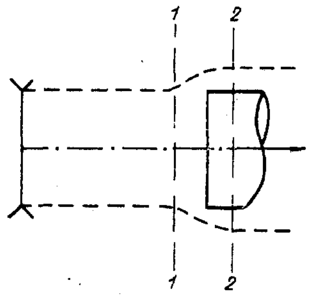

Fig. 2

where F = F(t, x), x is the coordinate along the symmetry axis of the jet, ρ is the density, v is the velocity, p is the pressure, and S is the entropy. According to the assumption that the quasisteady flow in the subsonic region after the central shock is upset only by entropy waves, we assume that the pressure depends only on the time, p = p(t), ∂p/∂x = 0. Then the general solution of the system (1.1) has the form

$$\mathscr{F}(v, \lambda) = 0,\ \lambda = vt - x,\ S = S(\lambda), \qquad (1.2)$$
$$p = p(t),\ p^{1/k}F(\mathscr{F}_v - \mathscr{F}_\lambda t) = \beta(\lambda),$$

where $\mathscr{F}$, S, p, and β are arbitrary functions of their arguments and k is the specific heat ratio. It follows from (1.2) that the characteristics λ = const are straight lines forming convergent or divergent bundles. If all the characteristics of a bundle pass through a common point (usually outside the flow field), we call such an entropy wave centered. Let a wave be centered at a point with coordinates (x_i, t_i). Then at this point the first relation (1.2) cannot be solved explicitly for the velocity v, because characteristics with different velocity values converge at the given point. Consequently,

$$\mathscr{F}_v\Big|_{\substack{x=x_i\\t=t_i}} + \mathscr{F}_\lambda\Big|_{\substack{x=x_i\\t=t_i}} t_i = 0.$$

Hence

$$v = (x - x_i)/(t - t_i), \qquad (1.3)$$

and the last relation (1.2) is rewritten

$$p^{1/k}F(1 - t/t_i) = \beta(\lambda). \qquad (1.4)$$

Thus, relations (1.3) and (1.4) determine a centered wave.

We now consider the special case in which the cross section depends only on the time, F = F(t), i.e., the generatrices of the cylindrical walls of the channel are parallel to the symmetry axis. It follows from the last relation (1.2) for F = F(t) that the velocity v obeys Eq. (1.3) and β ≡ const, i.e., in this channel the entropy wave is a special case of a centered wave.

§2. Without reiterating the arguments set forth in [3], we assume that the wake ends in an abrupt change of flow geometry and that the mass of the gas in the evolving wake is maintained mainly in its tail part, predominantly by gas flowing across the central shock (Fig. 3). The region of abrupt variation of the flow parameters may be likened to a certain fictitious discontinuity, the set of parameters subject to discontinuity including the cross section of the flow. We presume in keeping with our earlier assumptions that the pressure remains intact at such a discontinuity. Inasmuch as the variation of the parameters is sudden, but strictly speaking continuous, we regard the entropy as equal on either side of the shock. Consequently, the density is also continuous in the main flow, but in the wake it has a different value. The energy conservation principle is satisfied automatically under these assumptions, and for the two mechanical laws (conservation of mass and momentum) we can write the equations

$$F_1\rho(v_1 - N) + \rho_c(F_2 - F_1)(D - N) = \rho(v_2 - N)F_2, \qquad (2.1)$$
$$F_1\rho(v_1 - N)^2 + \rho_c(F_2 - F_1)(D - N)^2 = \rho(v_2 - N)^2F_2,$$

in which the subscripts refer to the corresponding cross sections (see Fig. 3), ρ is the density in those cross sections, N is the translational velocity of the tail of the wake, ρ_c is the density in cross section 1–1 of the wake, and D is the translational velocity of the

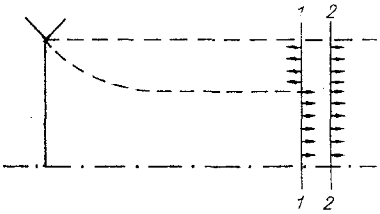

Fig. 3

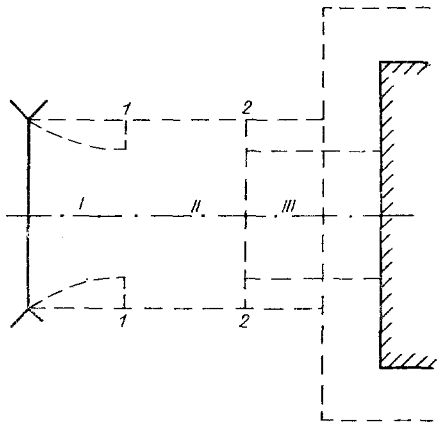

Fig. 4

central compression shock (all particles of the wake move with this velocity at time t). From (2.1) we obtain

$$N = \frac{v_2 (v_2 - D) - f (v_1 - D) v_1}{v_2 - D - f (v_1 - D)}, \tag{2.2}$$

where $f = F_1/F_2$ and D is computed at the same time at which the fictitious (conditional) discontinuity is analyzed.

§3. The evolution of the wave structure of the nonsteady jet zone within the limits of one period can now be described according to the following scheme. Suppose that at a certain intermediate position of the central shock a conditional discontinuity occurs in the parameters of the gas, its decay causing that shock to move in the direction away from the barrier and toward the nozzle, with a certain finite velocity $D_0 < 0$. The shock velocity decreases with time and vanishes at a certain instant, after which the shock begins to move in the opposite direction toward the barrier. The strength of the central shock decreases monotonically. Under these conditions a wake can develop after the triple point. If prior to decay a wake existed after the shock branch point, then at the instant of decay it separates from the shock configuration and is entrained by the flow, moving with the velocity of the latter.

Thus, after a small time interval following decay the flow pattern shown schematically in Fig. 4 emerges. The gas stream between the moving central shock and the barrier can divide into three parts. The first part is between the front of the central shock and cross section 1—1, where the new evolving wake ends (region I), the second is between cross sections 1—1 and 2—2 at the end of the separated flow-entrained wake (region II), and finally region III is the part between cross section 2—2 and the conditional barrier-simulating discontinuity. The boundary with the high-pressure flow across the two curvilinear shocks may be regarded in the given scheme as practically rectilinear and parallel to the jet axis. Consequently, in region III, where there is no wake, $F_2 \neq F(t)$, and the flow is a special case of a centered wave with $\beta = \beta_2 = \text{const}$. In region III it is the mass of the gas entrained in the wake, rather than the shape of the separated wake, that is significant. In this zone, therefore, it may also be assumed that the wake is bounded by cylindrical surfaces with rectilinear generatrices, i.e., $F_3 = F_3(t)$, and the entropy wave is centered with $\beta = \beta_3 = \text{const}$. In region I the inner boundary of the wake is essentially curvilinear. To take this fact into account we must augment the arbitrariness of the solution by one more arbitrary function of one argument, i.e., here the flow can also be described by a centered wave, but with $\beta = \beta_1(\lambda) \neq \text{const}$. As the wake in region I evolves it attains the cross section where the barrier is situated, i.e., wave I encompasses the entire region between the central shock and the barrier. If the wake is sufficiently well developed at this time, the annular gap between the outer high-pressure flow and the edge of the barrier is filled with the gas forming the wake, inducing complete flow stagnation immediately in front of the barrier. The lower boundary of the wake, i.e., the contact discontinuity there, extends beyond the surface of the barrier. These two facts (complete flow stagnation in front of the barrier and extension of the contact discontinuity beyond the surface of the barrier) have been stated repeatedly in experiments, with a certain astonishment, because it is customarily assumed that one contact discontinuity exists behind the shock branch point, making its emergence beyond the surface of the barrier seem unnatural.

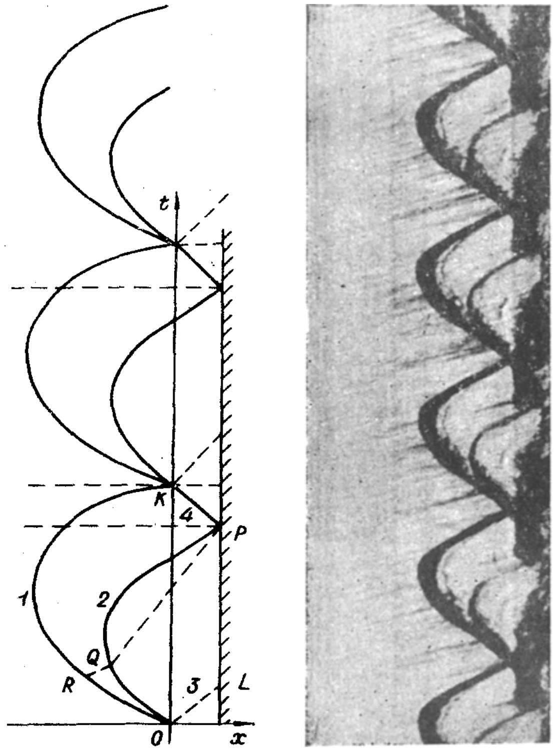

Fig. 5

The stagnation of the flow in front of the barrier can be simulated by a shock wave generated in front of the barrier (line 4 in Fig. 5). At a certain instant this shock encounters the central shock. A conditional discontinuity occurs in the parameters of the gas, its decay setting up the proper conditions for repetition of the cycle. At the instant of decay the wake separates from the triple shock configuration, and the proper conditions are set up for inception of a new wake. The separated wake is entrained by the flow, moving with the velocity of the latter, i.e., the closing cross section of this wake (2–2 in Fig. 4) is a contact discontinuity, which is separated upon decay. The entropy wave behind the shock emanating from the barrier is centered in the cross section where the barrier is situated, at the point where the contact discontinuity arrives. Before the arrival of this discontinuity the velocity in the cross section in front of the barrier is equal to zero and abruptly acquires a finite value at the instant of arrival of the contact discontinuity. On the right side of Fig. 5 is shown an experimental streak photograph of the process.* Fair qualitative and quantitative agreement obtains between the calculated and experimental patterns.

§4. Let us denote by l the distance from the nozzle orifice to the barrier. Let x_c be the analogous distance to the cross section in which successive decays of the conditional discontinuities take place. We assume that in the x, t plane the point O(0, 0) corresponds to a certain intermediate decay. In Fig. 5, line 1 represents the path of the central shock front, line 2 the path of the "tail" of the evolving wake, and line 3 the path of the contact discontinuity. We now derive equations for the main characteristics of the cycle. In region I the velocity is

$$v_1 = (x - x_1)/(t - t_1). \tag{4.1}$$

Let D be the translational velocity of the shock. Assuming that the distributions of all the parameters along the axis of the free jet are given, we use the compatibility conditions at the shock front and expression (4.1) to deduce from the equation $dx/dt = D$ a nonlinear first-order ordinary differential equation in the function $x = x_B(t)$, describing the path of the central shock in the x, t plane. To obtain finite relations we invoke the hypersonic approximation, since the Mach number downstream from the shock is usually large. We write the mass conservation principle for a normal shock in the vicinity of the symmetry axis:

$$\rho(v - D) = \rho_1(v_1 - D)$$

*Obtained by A. P. Petrov at the Institute of Theoretical and Applied Mechanics, Siberian Branch, Academy of Sciences of the USSR.

(ρ is the density, v is the downstream velocity, and v_1 is the upstream velocity relative to the shock); for large Mach numbers $\rho_1/\rho \approx (k+1)/(k-1)$, $v \approx v_m$, where v_m is the maximum steady-flow velocity. Consequently,

$$D \approx \frac{k+1}{2} v_1 - \frac{k-1}{2} v_m \tag{4.2}$$

or

$$\frac{\partial x}{\partial t} = \frac{k+1}{2}\frac{x-x_1}{t-t_1} - \frac{k-1}{2} v_m.$$

Hence,

$$x = x_B(t) = x_1 - v_m(t_1 - t) + c(t_1 - t)^{(k+1)/2} \tag{4.3}$$

(c is an arbitrary constant). The shock velocity at an arbitrary time is

$$D = \frac{dx_B}{dt} = v_m - \frac{k+1}{2} c(t_1 - t)^{\frac{k-1}{2}}. \tag{4.4}$$

Consider the motion of the "tail" of the evolving wake (cross section 1—1 in Fig. 4 or line 2 in Fig. 5). Let F be the area of the central shock. By the postulated rectilinearity of the boundary of the "outer" high-pressure flow $F_2 = F$. Writting expression (1.4) for the cross section immediately after the central shock and for cross section 2—2 (see Fig. 3) in the "tail" of the wake and then dividing the first result by the second, we obtain

$$\frac{\beta_1(\lambda)}{\beta_2} = \frac{1 - \frac{\bar{t}(\lambda)}{t_1}}{1 - \frac{t(\lambda)}{t_2}}, \tag{4.5}$$

where $t(\lambda)$ is the inverse of the function $\lambda(t)$ and $\bar{t}(\lambda)$ is the same for cross section 2—2,

$$\lambda(t) = \frac{x_B(t) - x_1}{t - t_1} t - x_B(t) = \frac{x_B(t)t_1 - tx_1}{t - t_1}.$$

Inasmuch as β_2 = const, relation (4.5) determines the function $\beta_1(\lambda)$. Forming the ratio of Eq. (1.4) for cross sections 1—1 and 2—2 (see Fig. 3), we obtain

$$f = \frac{\beta_2}{\beta_1(\lambda)}\frac{1 - t/t_2}{1 - t/t_1}. \tag{4.6}$$

In light of (2.2), (4.5), and (4.6); the equation dx/dt = N represents a nonlinear first-order differential equation of rather complex structure for the fucntion $x = \chi(t)$, the numerical integration of which is made difficult by the fact that the equation contains a great many parameters. We can construct a simple approximate solution of this equation by utilizing additional information acquired in the course of calculation of the oscillatory cycle; it is required to determine the coordinates and state parameters for at least three points of the unknown path, which are denoted in Fig. 5 by the letters O, P, and Q. From expression (2.2) we can compute at these points the corresponding values of the velocity of the "tail" of the wake, N_0, N_p, and N_q. With the aid of a Lagrange interpolation polynomial we can approximately reconstruct N from these values:

$$N \approx \frac{(t_q - t)(t_p - t)}{t_q t_p} N_0 + \frac{t(t_p - t)}{t_q(t_p - t_q)} N_q + \frac{t(t_q - t)}{t_p(t_q - t_p)} N_p.$$

Here the indices refer to the variables at the corresponding points. Integration of the relation dx/dt = N yields the function $\chi(t)$. More precisely,

$$x = \chi(t_1, t_2, t_q, t_p, x_C, t), \tag{4.7}$$

where the parameters contained by this function are listed.

In region III the velocity is

$$v_3 = (x - x_3)/(t - t_3), \tag{4.8}$$

and at the point L where the closing cross section of the separated wake arrives the velocity of the main flow changes abruptly, i.e., at this point there must be a centered wave with the velocity distribution (4.8). Therefore,

$$x_3 = l - x_C, \; t_3 = (l - x_C)/v_{3k}, \tag{4.9}$$

where the velocity v_{3k} on the last characteristic of region III is equal to the velocity after decay of the discontinuity upstream from the central shock moving with velocity D_0.

At time $t = t_p$ the evolving wake attains the barrier, complete stagnation takes place on the part of the flow across the central shock, and a reflected shock is formed, moving away from the barrier. According to the compatibility conditions for this shock its translational velocity has the following form on the assumption that it is relatively weak:

$$\frac{dx}{dt} = \frac{\frac{v_1 + v_3'}{2} - \frac{4}{k-1} a_1 \frac{v_1 - a_1}{v_1 - v_3'}}{1 - \frac{4}{k-1} \frac{a_1}{v_1 - v_3'}}. \tag{4.10}$$

Here v_1 and a_1 are the flow velocity and sound velocity ahead of the shock (flow in region I), and v_3' is the velocity in the region analogous to III but going over to the next cycle:

$$v_3' = \frac{x - l + x_c}{t - t_3 - T},$$

where T is the cycle period. From (4.10) we obtain a nonlinear first-order differential equation for the path of the shock front. In all streak photographs of the processes the path of the shock front is practically rectilinear, and the lifetime of this shock is small. To simplify the notation, therefore, we replace its velocity by the average value $(N_p + N_k)/2$, where N_p is the velocity of the reflected shock at the instant of its inception and N_k is the same velocity at the instant of encounter with the central shock. Both velocities are computed according to (4.10). Now the path of the shock front is approximately described by the equation

$$x = l - x_c + \frac{N_k + N_p}{2}(t - t_p). \tag{4.11}$$

§5. We now discuss a sequence of computational procedures on the basis of the qualitative scheme described above. This sequence rests on the exact solution of the boundary-value problems within the context of the stated assumptions. At the instant just before encounter of the central shock with the reflected shock moving according to the law (4.11) the postshock pressure p_{1k} is computed according to the well-known expression

$$\frac{p_{1k}}{p(x_c)} = \frac{2k}{k+1}\left[\frac{v(x_c) - D_1}{a_1(x_c)}\right]^2 - \frac{k-1}{k+1}, \tag{5.1}$$

in which $v(x_c)$, $a(x_c)$, and $p(x_c)$ are specified functions characterizing the distributions of the flow velocity, sound velocity, and pressure along the axis of the free jet (see, e.g., [5]) and D_1 is the translational velocity of the central shock at time $t = T$. According to (4.4),

$$D_1 = v_m - \frac{k+1}{2} c (t_1 - T)^{\frac{k-1}{2}}.$$

Let S_{1k} and a_{1k} be the entropy function and sound velocity computed at the same time upstream from the central shock:

$$S_{1k} = \frac{p_{1k}}{p(x_c)}\left[\frac{\rho(x_c)}{\rho_{1k}}\right]^k \approx \frac{p_{1k}}{p(x_c)}\left(\frac{k-1}{k+1}\right)^k, \quad \frac{a_{1k}}{a(x_c)} \approx \sqrt{\frac{p_{1k}}{p(x_c)} \frac{k-1}{k+1}}. \tag{5.2}$$

We use expression (4.10) to find the velocity,

$$N_k = \frac{\frac{v_{1k} + v_{3k}}{2} - \frac{4}{k-1} a_{1k} \frac{v_{1k} - a_{1k}}{v_{1k} - v_{3k}}}{1 - \frac{4}{k-1} \frac{a_{1k}}{v_{1k} - v_{3k}}}, \quad v_{1k} = \frac{x_1}{t_1 - T}.$$

Inasmuch as the velocity of the gas behind the reflected shock at the time of its inception is zero,

$$N_H = \frac{\frac{v_{1H}}{2} - \frac{4}{k-1} a_{1H} \frac{v_{1H} - a_{1H}}{v_{1H}}}{1 - \frac{4}{k-1} \frac{a_{1H}}{v_{1H}}}.$$

The procedure for computation of the sound velocity ahead of the reflected shock at time $t = t_p$ is no simple matter. It is necessary to determine the pressure p_{1H} and the entropy function at this point; p_{1H} is equal to the pressure upstream from the central shock at time $t = t_p$:

$$\frac{p_{1H}}{p(x_{cp})} = \frac{2k}{k+1}\left[\frac{v_{1p}-D_p}{a(x_{cp})}\right]^2 - \frac{k-1}{k+1}; \tag{5.3}$$

$$x_{cp} = x_c + x_p; \quad v_{1p} = \frac{x_p - x_1}{t_p - t_1}; \quad D_p = v_m - \frac{k+1}{2} c(t_1 - t_p)^{(k-1)/2}; \tag{5.4}$$

$$x_p = x_B(t_p) = x_1 - v_m(t_1 - t_p) + c(t_1 - t_p)^{(k+1)/2}. \tag{5.5}$$

The entropy function at point P is equal to the value of this function at point R behind the central shock (see Fig. 5), where the broken line RQP is the path of a particle arriving at point P. The equations for the segments of this line have the form

$$x_r - x_q = v_r(t_r - t_q), \; v_r = (x_1 - x_r)/(t_1 - t_r); \tag{5.6}$$

$$l - x_c - x_q = v_q(t_p - t_q), \; v_q = (x_q - x_2)/(t_q - t_2); \tag{5.7}$$

where x_2 and t_2 are constants in the equation for the velocity in region II and

$$v_2 = (x - x_2)/(t - t_2).$$

Moreover, according to (4.3),

$$x_r = x_1 - v_m(t_1 - t_r) + c(t_1 - t_r)^{(k+1)/2}, \tag{5.8}$$

and from (4.7) we obtain

$$x_q = \chi(t_1, t_2, t_q, t_p, t_c). \tag{5.9}$$

These relations are sufficient for the determination of S_{1H} and a_{1H}:

$$S_{1H} = S(D_r), \quad D_r = v_m - \frac{k+1}{2} c(t_1 - t_r)^{(k+1)/2}, \tag{5.10}$$

$$a_{1H} = a(p_{1H}, S_{1H}).$$

From (4.11) and the condition $x = 0$ at $t = T$ on the path of the reflected shock we deduce

$$T = t_p - 2(l - x_c)/(N_k + N_H). \tag{5.11}$$

In the steady-state oscillatory regime the decay of a conditional discontinuity at points O or K(O, T) must produce only amplification of the central shock and the inception of a contact discontinuity, i.e., immediately after decay the pressure can be computed by two techniques. On the one hand, this pressure is the pressure after the instantaneously amplified central shock,

$$\frac{p_{3k}}{p(x_c)} = \frac{2k}{k+1}\left[\frac{v(x_c) - D_0}{a(x_c)}\right]^2 - \frac{k-1}{k+1}. \tag{5.12}$$

On the other hand, p_{3k} is equal to the pressure behind the reflected shock just prior to decay,

$$\frac{p_{3k}}{p_{1k}} = \frac{2k}{k+1}\left(\frac{v_{1k} - N_k}{a_{1k}}\right)^2 - \frac{k-1}{k+1}. \tag{5.13}$$

To calculate the values of β_2 and $\beta_1(\lambda_k) = \beta_k$ at point K before decay we make use of relations (1.4) and (4.5):

$$\beta_2 = \beta_1(0) = p_{3k}^{1/k} F_0, \quad \beta_k = \frac{1 - T/t_2}{1 - T/t_1}\beta_2. \tag{5.14}$$

§6. For the 30 unknown parameters of the cycle x_1, t_1, v_{1k}, v_{1p}, D_0, D_1, v_{3k}, c, T, t_2, t_q, t_p, D_p, x_c, t_3, p_{1k}, p_{3k}, β_2, β_k, S_{1k}, x_2, a_{1k}, a_{1H}, p_{1H}, x_p, x_r, t_r, x_q, S_{1H}, and D_r we obtain from the foregoing equations a closed system of relations (see Fig. 5). We write Eq. (4.1) at points K and P, relations (1.4), (4.2)-(4.4) at points O and K, and (4.7) at point P. To the resulting nine equations we add (4.9), (5.1), (5.2)-(5.14), and the condition $dF/dt = 0$ at $D = 0$. The total number of these equations is 30, i.e., the number of unknown parameters. Since the majority of these relations are explicitly solvable or are easily solved for the unknowns, the system is easily reduced to a system of five nonlinear equations in the unknowns x_c, t_1, t_p, t_q, and t_r. This system is too bulky to write out here. The system can be further simplified on the basis of partial linearization, which follows as a special result from the qualitative analysis carried out above.

§7. For the realization of the periodic process it is necessary that within a single cycle the central shock decay monotonically, so that the postshock velocity v_1 will increase monotonically. Consequently, the entropy wave in region I must be centered at a point located in the first quadrant in the x, t plane, i.e., $x_1 > 0$, $t_1 > 0$. Also, since infinite values of the postshock velocity are inadmissible, $t_1 \geqslant T$. If $t_1 = T$, a gas sink will exist at the point K(0, T), and this is physically unrealistic. Therefore, the equality $t_1 = T$ (one of the roots of the system) is inadmissible. So we have as a necessary condition for realization of the oscillatory process the strict inequality $t_1 > T$. Inasmuch as the velocities at points O and L (Fig. 5) are identical, we have $v_{k_3} = x_1/t_1 = x_2/t_2 = (l - x_c)/t_3$, and all three entropy waves (in regions I, II, and III) are centered at points situated on the single line passing through points O and L. The area of the central shock at the beginning (point O) and at the end (point K) of the cycle is the same and equal to F_0. From (1.4) we obtain

$$\beta_k = p_{1k}^{1/k} F_0\left(1 - \frac{T}{t_1}\right).$$

From this result, using (5.14), we obtain

$$\frac{p_{3k}^{1/k}}{p_{1k}^{1/k}} = (1 + \Delta p)^{1/k} = 1 - \frac{T}{t_2}, \tag{7.1}$$

where Δp is the relative intensity of the reflected shock before decay. Assuming that the reflected shock is weak, we obtain from (7.1) correct to second-order small quantities

$$t_2 \approx -kT/\Delta p,$$

i.e., t_2 must be a large negative number. The entropy wave in region II is centered in the third quadrant in the x, t plane. The quantities t_p and t_q enter into the analytical relations in the form $1 - t_p/t_2$ and $1 - t_q/t_2$. Linearization is admissible under the assumptions $t_p/t_2 \ll 1$ and $t_q/t_2 \ll 1$. The smaller the ratio T/t_1 relative to unity the more slowly will the velocity upstream from the central shock vary and the larger will be the period T of the oscillatory cycle. For low frequencies $T/t_1 \ll 1$, and the analogous statements pertinent to this latter inequality are possible.

The domain of existance of the roots corresponds to the domain of existence of the self-sustaining oscillatory process. The proposed mathematical model has not been methodically tested. The published experimental results pertaining to the wave structure of the cycles are scant and are by and large of a qualitative nature.

LITERATURE CITED

1. B. G. Semiletenko, B. N. Sobkolov, and V. N. Uskov, "Characteristics of the unstable interaction of a supersonic jet with an infinite barrier," Izv. Sibirsk. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk Ser. 13, No. 3 (1972).
2. A. G. Golubkov, B. K. Koz'menko, V. A. Ostapenko, and A. V. Solotchin, "Interaction of a supersonic underexpanded jet with a plane finite barrier," Izv. Sibirsk. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, Ser. 13, No. 3 (1972).
3. V. G. Dulov, "Motion of a triple shock configuration with the formation of a wake behind the branch point," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1973).
4. A. G. Golubkov and V. G. Dulov, "Simulation of the influence of a barrier in the path of a supersonic jet by a conditional discontinuity of the flow parameters," Izv. Sibirsk. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, Ser. 3, No. 1 (1975).
5. V. G. Dulov, "Flow models approximating the properties of supersonic jet flows," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1976).